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ABSTRACT

The use of the two-stage least squares (2 SLS) procedure for estimating nonrecursive social science models is often impractical when multiple feedback linkages are required. This is because 2 SLS is extremely sensitive to multicollinearity. The standard statistical solution to the multicollinearity problem is a biased, variance reduced procedure known as ridge regression (RR). However, RR is a stochastic solution when it should be nonstochastic, and does not meet the boundary condition; thus, a new ridge technique called normalization ridge regression (NR) is proposed. When NR is used in an analogous fashion to 2 SLS it should theoretically provide a superior solution to multiple feedback estimating problems than either 2 SLS or 2 SRR. All three techniques (2 SLS, 2 SRR, and 2 SNR) are used to estimate a multiple feedback model of school science achievement for a national probability sample of Finnish 14 year-olds. The 2 SNR estimates were compared to both 2 SLS and 2 SRR estimates. In general, the 2 SNR results were the most consistent and confirmed the posited advantages of the biased, variance-reduced 2 SNR procedure over the other nonrecursive estimating methods. (Author/DWH)

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NORMALIZATION RIDGE REGRESSION IN PRACTICE II: THE ESTIMATION OF MULTIPLE FEEDBACK LINKAGES*

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ABSTRACT

The use of the two-stage least squares (2SLS) procedure for estimating nonrecursive social science models is often impractical when multiple feedback linkages are called for. This is because 2SLS is extremely sensitive to multicollinearity. The standard statistical solution to the multicollinearity problem is a biased, variance reduced procedure known as ridge regression (RR). RR, however, is a stochastic solution when it should be nonstochastic, and does not meet the boundary condition; thus, a new ridge technique called normalization ridge regression (NR) is proposed. When NR is used in an analogous fashion to 2SLS it should theoretically provide a superior solution to multiple feedback estimating problems than either 2SLS or 2SRR. All three techniques (2SLS, 2SRR, and 2SNR) are used to estimate a multiple feedback model of school science achievement for a national probability sample of Finnish 14 year-olds. The 2SNR estimates were compared to both 2SLS and 2SRR estimates. In general, the 2SNR results were the most consistent and confirmed the posited advantages of the biased, variance-reduced 2SNR procedure over the other nonrecursive estimating methods.

The purpose of this paper is to demonstrate the utility of a new estimating method called two stage normalization ridge regression analysis (2SNR) for estimating feedback parameters in nonrecursive, nonorthogonal structural equation models.

In part one the problem is identified.

In part two a solution to the problem is proposed.

In part three the proposed solution is applied to a problem in social modeling; namely, a model of science achievement for a national probability sample of Finnish fourteen year-olds.

Herman Wold (1978) has recently drawn attention to the lack of fit between the complexity of contemporary social problems and the current state of social science methodology for investigating these problems. He points out that most statistical methods in use are traceable to the seminal work of Fisher (1925, 1935) and his followers; and that these methods were designed predominantly for the estimation of "simple" problems characterized by a high degree of information and prior insight. In contrast, today's social problems -- ecological imbalance, the arm's race, alcoholism and drug abuse, poverty amid affluence, world-wide inflation, youth unemployment and others -- tend to be complex in that numerous interrelationships between many variables are involved and prior insight into these problems is low. Thus, today's applied statistician

is faced with the formidable task of estimating what Wold labels "soft" models, many of which are interdisciplinary.

Some of the problems in multidisciplinary studies focus on change or developmental phenomena in living systems. Consequently the structural models of the change processes in a variety of disciplinary domains -- sociology, economics, political science and psychology -- call for positive feedback or reciprocal relationships among mutually causal processes. Positive feedback may be specified when examining the poverty-apathy cycle (sociology), the unemployment-deflation cycle (economics), the arms race (political science), or progressive drug addiction (psychology). Compensatory educational practices may involve efforts to reverse a vicious cycle of failure through the initiation of a positive feedback process as follows: A meaningful and achievable task is provided ... task achievement generates a sense of accomplishment promotive of ambition . . ambition promotes further striving for accomplishment ... the feedback is again positive ... the task performance goes well ... the pupil tries again ... In short, in the acquisition of of any desired skill, increased facility tends to enhance motivation which in turn facilitates the acquisition of further skill and so on.

PART ONE: THE PROBLEM

Those who have attempted to estimate nonrecursive feedback models will know from first hand experience how difficult it can be to interpret the feedback estimates. Usually a two-stage least squares (2SLS) estimating technique is used by the sociologist, though other methods include three stage least squares and maximum likelihood approaches. Though the discussion here is limited to the 2SLS method, feedback estimation can be problematic regardless of the estimation method used.

Though 2SLS performs well in terms of bias and mean square error; and though it is not exclusively sensitive to specification error, most analysts note its extreme sensitivity to multicollinearity (Christ, 1966, pp. 477-481; Intrilligator, 1978, pp.419-420; Johnston, 1972, p. 411; and King, 1974): a drawback which limits its practical utility. Multiple feedback estimation, or the estimation of models which call for two or more feedback linkages, is particularly problematic when using standard nonrecursive methods because the larger the number of nonrecursive parameters the more acute the multicollinearity problem.

Multicollinearity refers to the presence of highly inter-correlated independent variables in structural equation models, and its effect is to invalidate some of the basic assumptions on which least squares estimating methods are based (Vinod, 1978). Multicollinearity is a pervasive problem in social

modeling. It is not one of kind -- of existence or nonexistence -- but rather one of degree.

The harmful effects of multicollinearity produce estimates which are: (i) too large in magnitude, (ii) too sensitive to sampling error, and (iii) too sensitive to specification error. And such sensitivity results in (i) wrong signs on some coefficients, (ii) drastic changes in regression coefficients after minor data revision, (iii) conflicting conclusions from the usual significance tests, and (iv) inflated variance of the estimates. Thus, in the presence of acute multicollinearity, regression estimates are often erroneous and uninterpretable.

PART TWO: WHAT CAN BE DONE?

In order to try to cope with the problem it is desirable to know why standard methods of feedback estimation are sensitive to multicollinearity. The 2SLS model has more than one endogenous variable on the right-hand side. The solution calls for each of these right hand side variables to be replaced by a variable which corresponds to the estimates or predictions from the relevant reduced-form equation. Each reduced-form equation expresses a single endogenous variable in terms of the model's source variables or exogenous variables. The general idea is to "purify" the endogenous variables so that they are uncorrelated with the disturbance term in the equation.

In the first stage of the 2SLS technique, then, there will be as many regressions as there are right-hand side endogenous variables; that is each dependent variable in each regression will be one of the endogenous variables on the right-hand side of the structural equation, while in each regression the explanatory variables will be the exogenous variable set.

Consider a single nonrecursive equation of the general form:

$$y = \hat{a}_1\hat{Z}_1 + \hat{a}_2\hat{Z}_2 + \ldots + \hat{a}_k\hat{Z}_k + (b_1X_1 + \ldots + b_mX_m) + \alpha \quad (1)$$

where y is the dependent variable, the X's are the exogenous variables; the $\hat{Z}$'s are the estimated endogenous variables; α is the constant term; and the $\hat{a}$'s and b's are the estimated endogenous and exogenous variable coefficients.

Because the Z's are the right-hand side endogenous variables, at the first stage of estimation there will be as many regressions as there are Z's. In each regression the explanatory variables will be the X's which in Eq. (1) were placed in parentheses. The second stage of the procedure consists of a single regression on the structural equation ($y=\hat{a}_1\hat{Z}_1 + \ldots + \hat{a}_k\hat{Z}_k + \alpha$) where the right-hand side endogenous variables (the Z's) have been replaced by the predicted estimates (the $\hat{Z}$'s) generated in the first stage. See, for example, the text by Namboodiri et al. (1975, pp.513-517).

The reason that 2SLS is sensitive to multicollinearity is because estimated values are used for right-hand side variables at the second stage. Thus, only those parts of the variation that can be explained in terms of the exogenous variables in the model are being used. The rest of the observed variation for each endogenous variable is discarded. The variance which remains in each endogenous variable will, as a result, covary both with the exogenous variables and with the other estimated endogenous variables. This means that the predictors at the second stage of the estimating procedure are likely to be highly correlated; and, further, the larger the number of nonrecursive equations called for the higher the correlations among the predictor variables will be.

This source of multicollinearity is statistical. It enters through the diagonal elements of the inverse of the

correlation matrix of predictor variables ($tr(X'X)^{-1}$). The elements are the variance inflation factors (VIF's) of the $(X'X)^{-1}$ matrix. They constitute measures of the collective impact of the explanatory variables on the variance of the regression coefficient of each term (Marquardt and Snee, 1975, p. 4). The standard solutions to the multicollinearity problem developed by Hoerl and Kennard (1970) involve augmenting the diagonal of the normal equation matrix with a small positive quantity called, k, a constant, in order to generate regression coefficients with smaller variance. Thus, the smaller variance is obtained at the expense of introducing a modest degree of bias. The argument is that an estimator that has a small bias and a small variance is to be preferred to one that has no bias but a large variance. The technique is called ridge regression (RR).

The argument in favour of small bias and small variance (see, for example, Goldberger, 1964, p. 127) is strongly supported by Monte Carlo experiments in which the true regression coefficients are specified. In these experiments the estimated regression coefficients using RR and OLS are compared in terms of mean squared error (MSE); that is, in terms of the average squared distance between estimator and parameter. Vinod (1978, p. 121) reports sixteen Monte Carlo experiments from diverse fields. He states that "the superiority of RR over OLS is almost always noted."

Though RR goes much of the distance toward overcoming the multicollinearity problem, the procedure is not foolproof (see Bulcock et al. 1980, pp. 40-41, 47-50). Some of the dilemmas

of the RR approach stem from use of the MSE criterion. In theory, use of a new criterion called the variance normalization criterion copes with these dilemmas while retaining the advantages of RR (Bulcock et al. 1980). The preferred procedure called normalization ridge regression (NR) seems to be a consistently adequate estimating method even though it does not automatically give a solution which is always superior to that of any single RR method. NR may be regarded, therefore, as one of a family of solutions, none of which gives an automatic "best" solution to the estimating problem. Thus, although the Monte Carlo experimental evidence is impressive there is wide disagreement about the optimum RR method.

In the remainder of this paper a model with multiple feedback parameters is formulated and estimated. The feedback linkages are estimated using 2SLS, two-stage normalization ridge regression(2SNR) and six different two-stage ridge regression methods (2SRR). The ridge methods are those of Kasarda and Shih (1977), Hocking, Speed and Lynn (1976), Lawless and Wang (1976), Hoerl, Kennard and Baldwin (1975), Hoerl and Kennard's iterative method (1976), and Vinod's index of stability of relative magnitude (ISRM) method.

Most analysts recognize that it is not enough to compare parameter estimates. Thus, performance indices have been developed to facilitate comparisons between methods. Following Hoerl and Kennard (1970), Marquardt and Snee (1975) and McCabe (1978) comparisons will be made between (i) the biasing parameters, k; (ii) the inflated residual sums of squares, ϕ's;

(iii) the maximum variance inflation factors, Vmax; and (iv) the α-acceptability index. In addition (v) the degree of multicollinearity index, Dmax; (vi) the empirical sensitivity ratio, E.S.Ratio; and (viii) the relative reduction in variance, RVAR, will be used for comparative purposes.

For any estimating method it is desirable: (i) to reduce the Vmax to a value close to unity; (ii) to generate the maximum reduction of the relative degree of multicollinearity (Dmax) without introducing too large a bias (k); (iii) to minimize the sampling variance or E.S.Ratio; (iv) to reduce the total variance of the OLS estimates inflated by the presence of multicollinearity; and (v) to produce low, or other things being equal, acceptably low inflation in the residual sum of squares (ϕ_0).

PART THREE: APPLICATION OF THE PROPOSED SOLUTION TO A MODEL OF SCHOOL SCIENCE ACHIEVEMENT

3.1 Model Formulation

Consider a model of school science achievement for fourteen year-olds in which it is hypothesized:

(i) that both the student's linguistic competencies and literacy-numeracy skills affect school science achievement;

(ii) that linguistic competencies are developmentally prior to the acquisition of literacy-numeracy skills;

(iii) that over and above linguistic competencies and literacy-numeracy skills, a student's achievement in science will be a function of that student's stage of thinking;

(iv) That science achievement in open societies will be independent of social, regional and sexual differences; that is, the process of science achievement will be the same regardless of a student's home background, community of residence and sex; and

(v) that scientific competencies will be stimulated by positive attitudes towards science and vice versa; that is, science attitudes and science achievements will be mutually reinforcing or reciprocal.

Such a model is conceptualized in Figure 1. The variables are as follows:

FOCC Father's occupational status

PARED Aggregate level of father's and mother's education

SIBSZ	Number of children in family
SES	A socioeconomic status composite
SEX	Boy or girl
URBAN	Type of community in which the student's school is located
VERBAL	Verbal ability
THINK	Piagetian thinking stage variable
READ	Reading comprehension
CHEM	Achievement in chemistry
PHYS	Achievement in physics
SCATT	Science attitude

Figure 1 about here

Note the following: (i) that URBAN, VERBAL and READ are proxy variables standing for regional differences, linguistic competencies and literacy skills respectively; (ii) that SES, SEX and URBAN are the inequality variables; (iii) though a numeracy variable was specified (hypothesis #2), it was not included in the model (cf. Figure 1); (iv) science achievement is disaggregated into performances in chemistry and physics; and (v) that reciprocal relationships or multiple feedback linkages are specified between the three outcomes of the science achievement process -- CHEM, PHYS and SCATT.

3.2 2SLS Estimation

In Figure 2 the 2SLS results are presented for a national probability sample of Finnish 14 year-olds. The data were

obtained from the IEA data bank repository at the University of Stockholm. Obviously, the Figure 2 path coefficients are uninterpretable. They illustrate all the problems of acute multicollinearity. (i) The estimates are inflated. (ii) Several paths seem to have the wrong sign (compare the path coeficients with the zero-order relationships presented below). (iii) Several paths with large coefficients are not statistically significant, indicating unacceptably high standard errors. Before these uninterpretable results can be explained it is necessary to specify the model equations and examine their data matrices and eigenvalues.

Figure 2 about here

3.3 Model Equations and 2SNR Estimates

The SES and SCATT variables were constructed as principal component factors:

$$SES = w_1 FOCC + w_2 PARED + w_3 SIBSZ \quad (2)$$

$$SCATT = w_1 ATT1 + w_2 ATT2 + \ldots + w_6 ATT6 \quad (3)$$

where the w's are factor score coefficients.

There is one recursive equation:

$$READ = b_1 SES + b_2 SEX + b_3 VERBAL + b_4 URBAN + b_5 THINK + \alpha + \varepsilon \quad (4)$$

where the b's are regression coefficients, α is the constant term, and ε is the residual or error term.

There were three nonrecursive equations. In each case it was assumed that effects of verbal ability, sex and type of community were mediated via reading comprehension. In this

way, by assigning the VERBAL, SEX and URBAN variables to the exogenous variable set, the identification problem could be handled.

$$\begin{aligned} CHEM = {} & \hat{a}_2\hat{PHYS} + \hat{a}_3\hat{SCATT} + \hat{a}_4\hat{READ} + \hat{a}_5\hat{SES} \\ & + \hat{a}_6\hat{THINK} + (b_1 FOCC + b_2 PARED + b_3 SIBSZ + \\ & b_4 SEX + b_5 URBAN + b_6 VERBAL) + \alpha + \varepsilon \end{aligned} \tag{5}$$

$$\begin{aligned} PHYS = {} & \hat{a}_1\hat{CHEM} + \hat{a}_3\hat{SCATT} + \hat{a}_4\hat{READ} + \hat{a}_5\hat{SES} + \\ & \hat{a}_6\hat{THINK} + (b_1 FOCC + b_2 PARED + b_3 SIBSZ + \\ & b_4 SEX + b_5 URBAN + b_6 VERBAL) + \alpha + \varepsilon \end{aligned} \tag{6}$$

$$\begin{aligned} SCATT = {} & \hat{a}_1\hat{CHEM} + \hat{a}_2\hat{PHYS} + \hat{a}_4\hat{READ} + \hat{a}_5\hat{SES} + \\ & \hat{a}_6\hat{THINK} + (b_1 FOCC + b_2 PARED + b_3 SIBSZ + \\ & b_4 SES + b_5 URBAN + b_6 VERBAL) + \alpha + \varepsilon \end{aligned} \tag{7}$$

The correlation matrix for the variables in the science achievement model is presented in Table 1. The correlation matrices and X'y eigenvalues of the second stage estimating procedure for equations 5 through 7 are presented in Tables 2 to 4. Since the expected squared distance of the least squares coefficient $\hat{\beta}$ from $\beta = \Sigma\lambda_i^{-1}/p$ the distance for equation 5 (CHEM) is about 30 times what it would be for an orthogonal system. For equations 6 and 7 the distances are over 50 and 800 times greater respectively than would be the case for orthogonal data sets. The degree of multicollinearity for equations 5 through 7 is far too high in each instance.

Tables 1 thru 4 about here

In Figure 3 the results of the preferred two-stage normalization ridge regression (2SNR) analysis are presented. The evidence suggests that the preference for an estimator with a small bias and small variance to one with no bias but a large variance is advantageous. In other words, if multicollinearity, and only multicollinearity, can be reduced without introducing too much bias as was accomplished by the 2SNR technique, the limitations of the standard 2SLS estimating method can be coped with.

Figure 3 about here

3.4 Performance Indices

The 2SLS and 2SNR estimates together with those for six 2SRR methods are presented in Tables 5 through 7. Though it seems as if the less erratic 2SNR results are to be preferred over the 2SLS estimates the important question is, "How do you know?" There is no single answer to this question. The bias depends on the population coefficients and, therefore, cannot be obtained. No single performance measure such as the MSE can be used to evaluate the Table 5 through 7 estimates. The results of seven performance indices for each estimating method are, therefore, presented in Tables 8 through 10. These are summarized, in turn, in Table 11.

Tables 5 through 11 about here

In general, the Kasarda-Shih method was too conservative and, therefore seldom generated results which differed from the OLS results. The Hoerl-Kennard-Baldwin results were also on the conservative side. Both had the same limitations as 2SLS. Vinod's results were also questionable because they generated more than one minimum point even though in theory there should only be one optimal k giving minimum MSE. Of the remaining four methods the normalization technique proved to be the most consistent across the three equations. Even so, for the Eq. (7) results Hoerl and Kennard's iterative method marginally outperformed the 2SNR method -- compare the performance rankings for Vmax, Dmax, E.S.Ratio and RVAR in Table 11.

PART FOUR: CONCLUDING REMARKS

1. The purpose of this paper was to examine the operation of ridge regression in practice. In particular, the utility of 2SNR as a technique for coping with the estimation of multiple feedback linkages in structural equation models was assessed.

2. This task called for the estimation of highly collinear input data at the second stage in the estimation of a multiple feedback model. It was shown that 2SLS as the standard procedure was unequal to the task. The 2SLS estimates of the right-hand side endogenous variable matrix illustrated all the problems of acute multicollinearity -- wrong signs, inflated estimates, and large standard errors. Such results would support erroneous conclusions from the usual tests of statistical significance.

3. The question posed, then, was whether the inherent multicollinearity problem in nonrecursive model estimation could be handled using the standard biased, variance-reduced approaches in the Hoerl and Kennard tradition. Unfortunately RR approaches are not problem-free. Thus, a new procedure called normalization ridge regression was proposed. In theory, normalization ridge regression enjoys the advantages of RR while at the same time overcoming several of RR's disadvantages. In the application of 2SRR and 2SNR to the science achievement model the normalization ridge regression results were compared to those from six different RR methods. It was shown that 2SNR did not automatically generate better estimates in terms of

performance criteria than 2SRR; nevertheless, the 2SNR technique was the most consistent. Thus, for the multiple feedback science achievement model the 2SNR estimates were the ones to be preferred.

4. An examination of the substantive 2SNR results shows that most hypotheses were supported. One result, however, defied interpretation. Achievement in chemistry was dependent on primary linguistic skills and secondary resources (reading comprehension) but this was not found to be the case with physics. The refutation of the basic skills resource hypothesis in the case of physics is puzzling. It would be interesting to see whether the result could be replicated for Swedish or English IEA science achievement data. Support for the science learning system hypothesis was, therefore, mixed. Reciprocal relationships between science attitudes and chemistry were noted but the hypothesized reciprocal relationships between science attitude and physics were non-existent.

5. As hypothesized the thinking stage variable was a significant determinant of both physics and chemistry (especially physics).

6. The equity hypotheses were only partially supported. First, the Finnish science curriculum was sex biased in favour of boys. This was because the sex variable was relatively orthogonal to everything except physics and chemistry. See Table 1. Thus, even if controls were placed on all other variables in the model the sex differences in physics and chemistry would still be present. Second, the regional hypothesis was supported.

There was no evidence (see Table 1) that science achievement was a function of regional inequalities, or more specifically, the type of community in which a student's school was located. Third, though home background (SES) effects on reading, chemistry and science attitude were negligible, the effect on physics was negative. Such a finding -- that children from modest social backgrounds may perform marginally better in physics that more socially privileged children (ceteris paribus) -- was unanticipated. According to these results physics is no respecter of social class.

7. Finally, it is emphasized that the purpose of this analysis was to demonstrate the utility of 2SNR as a method for coping with the multicollinearity problem in nonrecursive model estimation. On the basis of the evidence it seems that this new technique may be a promising procedure for coping with (but not solving) the multicollinearity problem in nonrecursive model estimation. More research is desirable in order to gauge the advantageous properties of 2SNR. Monte Carlo simulation experiments would probably throw more light on the matter.

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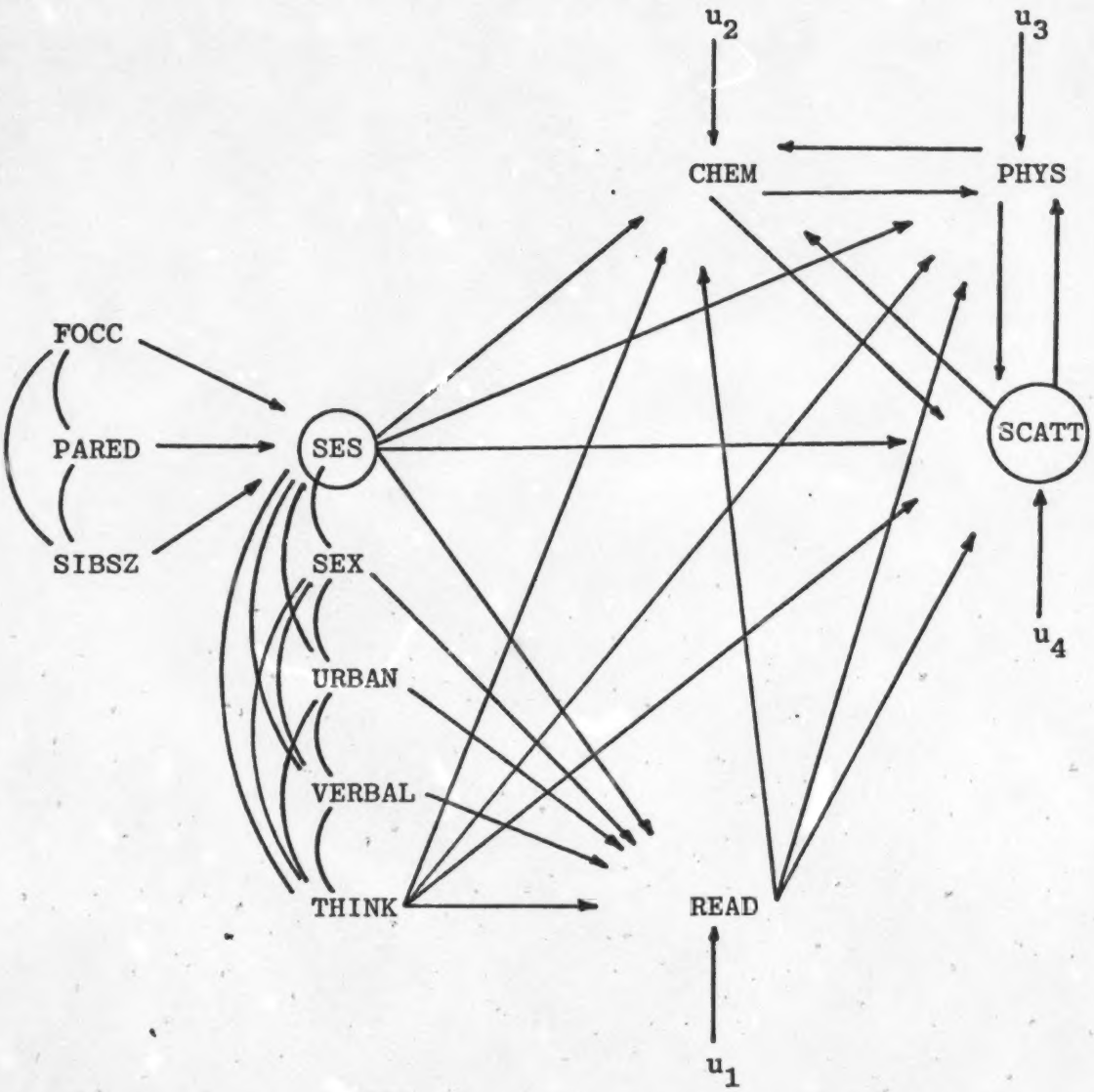
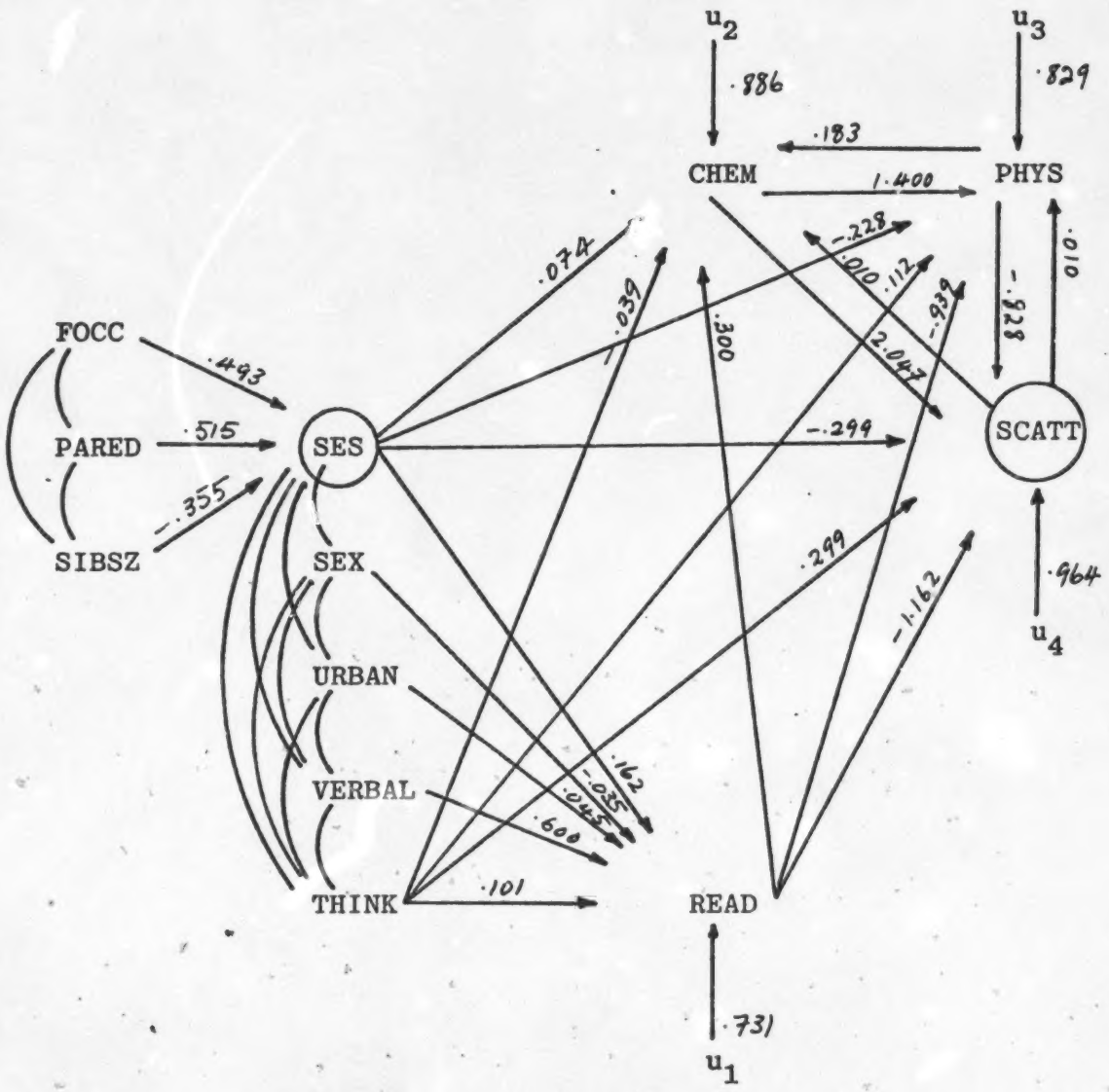
FIGURE 1. Conceptual Model of School Science Achievement



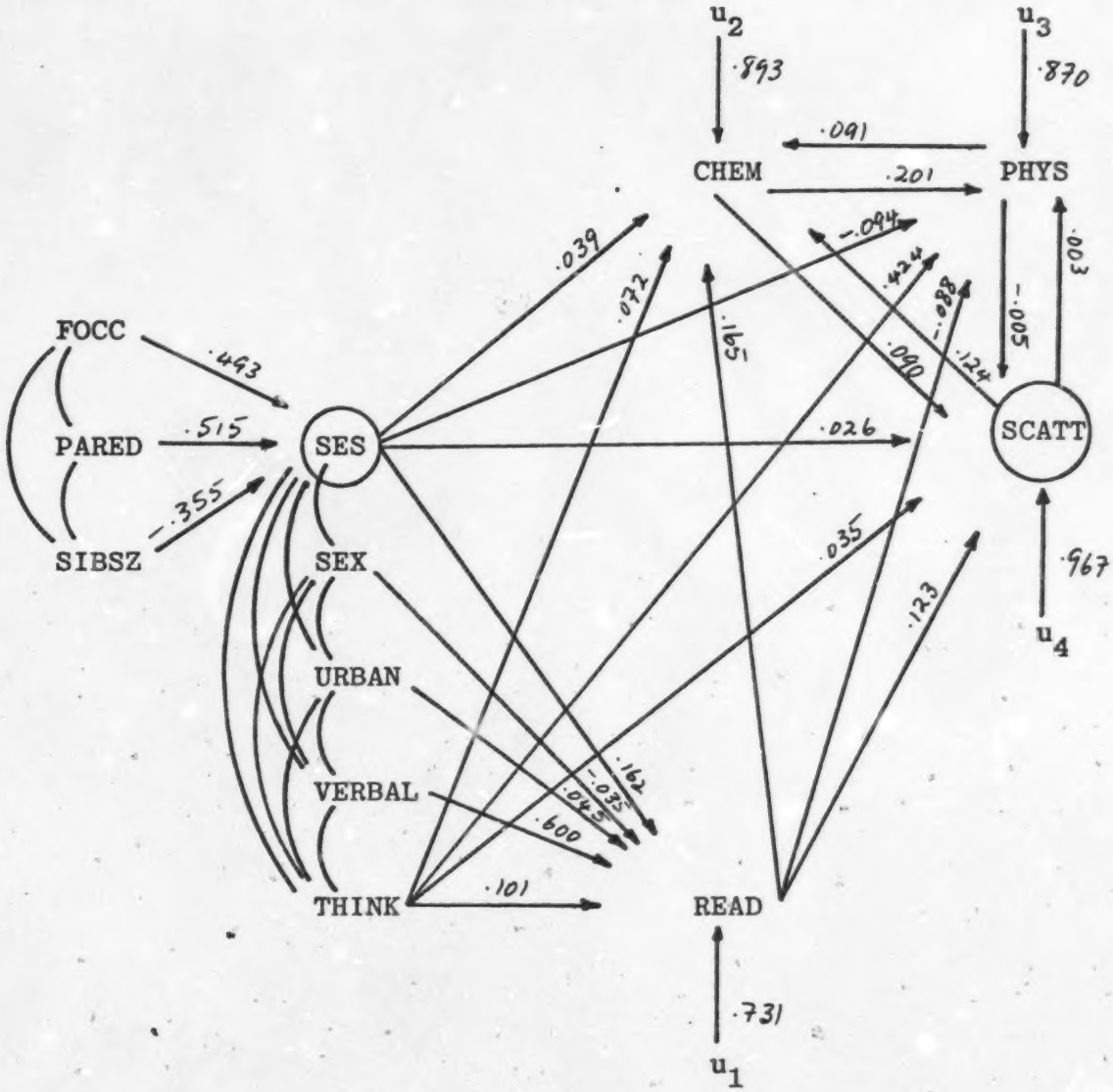
Mnemonics: FOCC = father's occupational status; PARED = parental education; SIBSZ = number of children in family; URBAN = type of community (proxy for region); VERBAL = verbal ability (proxy for linguistic resources); THINK = thinking stage variable; READ = reading comprehension (proxy for literacy); CHEM = achievement in chemistry; PHYS = achievement in physics; SCATT = student attitude toward science.

FIGURE 2. 2SLS Estimates



Mnemonics: FOCC = father's occupational status; PARED = parental education; SIBSZ = number of children in family; URBAN = type of community (proxy for region); VERBAL = verbal ability (proxy for linguistic resources); THINK = thinking stage variable; READ = reading comprehension (proxy for literacy); CHEM = achievement in chemistry; PHYS = achievement in physics; SCATT = student attitude toward science.

FIGURE 3. 2SNR ESTIMATES



Mnemonics: FOCC = father's occupational status; PARED = parental education; SIBSZ = number of children in family; URBAN = type of community (proxy for region); VERBAL = verbal ability (proxy for linguistic resources); THINK = thinking stage variable; READ = reading comprehension (proxy for literacy); CHEM = achievement in chemistry; PHYS = achievement in physics; SCATT = student attitude toward science.

Table 1. Correlation Matrix, Means and Standard Deviations for the Science Achievement Model (N=1660)

Variables	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}	$\bar{X}$	SD
X_1 FOCC	1.000												2.76	.966
X_2 PARED	.449	1.000											5.73	1.38
X_3 SIBSZ	-.167	-.230	1.000										3.55	1.31
X_4 SES	.780	.816	-.556	1.000									.055	1.00
X_5 SEX	-.064	-.018	.067	-.065	1.000								1.50	.500
X_6 VERBAL	.170	.243	-.161	.266	.048	1.000							16.92	9.57
X_7 URBAN	-.154	-.241	.159	-.256	.008	-.075	1.000						3.44	2.11
X_8 THINK	.093	.110	-.061	.124	-.122	.152	-.057	1.000					2.32	.892
X_9 READ	.240	.308	-.137	.325	-.028	.654	-.047	.214	1.000				27.55	10.70
X_{10} CHEM	.205	.250	-.123	.273	-.132	.404	-.042	.193	.451	1.000			3.87	3.54
X_{11} PHYS	.177	.182	-.103	.217	-.409	.346	-.033	.239	.432	.428	1.000		7.74	4.11
X_{12} SCATT	.100	.142	-.101	.158	-.039	.240	-.032	.066	.311	.218	.237	1.000	.055	1.01

Table 2. Correlation Matrix and Eigenvalues of X'X for Equation 10 (CHEM) (N=1660, Dc=.992)

Variables	X_{11} PHYS	X_{12} SCATT	X_9 READ	X_4 SES	X_8 THINK	X_{10} CHEM
X_{11} PHYS	1.000					
X_{12} SCATT	.766	1.000				
X_9 READ	.711	.974	1.000			
X_4 SES	.389	.598	.478	1.000		
X_8 THINK	.953	.873	.810	.573	1.000	
X_{10} CHEM	.397	.454	.445	.273	.431	1.000
λ_i [a]	.0117	.0169	.3894	.6759	3.906	

a. $\Sigma\lambda_i^{-1} = 148.945$

Table 3. Correlation Matrix and Eigenvalues of X'X for Equation 11 (PHYS) (N=1660. Dc=.997)

Variables	X_{10} CHEM	X_{12} SCATT	X_9 READ	X_4 SES	X_8 THINK	X_{11} PHYS
X_{10} CHEM	1.000					
X_{12} SCATT	.979	1.000				
X_9 READ	.960	.974	1.000			
X_4 SES	.588	.598	.478	1.000		
X_8 THINK	.930	.873	.810	.573	1.000	
X_{11} PHYS	.479	.428	.397	.217	.532	1.000
λ_i [a]	.0048	.0139	.2000	.6197	4.162	

a. $\Sigma\lambda_i^{-1} = 287.130$

Table 4. Correlation Matrix and Eigenvalues of X'X for Equation 12 (SCATT) (N=1660, Dc=.9999)

Variables	X_{10} CHEM	X_{11} PHYS	X_9 READ	X_4 SES	X_8 THINK	X_{12} SCATT
X_{10} CHEM	1.000					
X_{11} PHYS	.857	1.000				
X_9 READ	.960	.711	1.000			
X_4 SES	.588	.389	.478	1.000		
X_8 THINK	.930	.953	.810	.573	1.000	
X_{12} SCATT	.259	.202	.257	.158	.231	1.000
λ_i [a]	.000231	.0189	.3376	.6793	3.964	

a. $\Sigma\lambda_i^{-1} = 4385.129$

Table 5. Standardized Partial Regression Coefficients, Equation 10 (CHEM), Eight Estimating Methods[a]

Estimating Method	X_{11} PHYS	X_{12} SCATT	X_9 READ	X_4 SES	X_8 THINK	R^2
1. OLS	.183	.010	.300	.074	-.039	.215
2. Normalzn.	.091	.124	.165	.039	.072	.202
3. Kasarda	.132	.085	.226	.052	.018	.212
4. Hocking	.144	.070	.242	.057	.004	.213
5. H.K.& B.	.125	.093	.216	.049	.026	.212
6. L.& W.	.139	.077	.234	.054	.011	.213
7. H.K. Iter	.111	.109	.194	.044	.044	.210
8. Vinod	.092	.124	.168	.039	.070	.203
(2-values)	.075	.096	.101	.044	.080	.129

a. The underlined coefficients are not statistically significant at the .05 level.

Table 6. Standardized Partial Regression Coefficients, Equation 11 (PHYS), for Eight Estimating Methods[a]

Estimating Method	X_{10} CHEM	X_{12} SCATT	X_9 READ	X_4 SES	X_8 THINK	R^2
1. OLS	1.400	.010	-.939	-.228	.112	.312
2. Normalzn	.201	.003	-.088	-.094	.424	.243
3. Kasarda	1.192	-.013	-.793	-.210	.198	.308
4. Hocking	.662	-.053	-.433	-.164	.404	.296
5. HK&B	1.244	-.008	-.829	-.214	.177	.309
6. L. & W.	.692	-.052	-.453	-.167	.393	.297
7. H.K. Iter	1.210	-.012	-.806	-.211	.191	.308
8. Vinod	.245	-.018	-.138	-.112	.467	.261
(2-values)	.112	-.064	.043	-.008	.186	.152

a. The underlined coefficients are not statistically significant at the .05 level.

Table 7. Standardized Partial Regression Coefficients, Equation 12 (SCATT), for Eight Estimating Methods[a]

Estimating Method	X_{10} CHEM	X_{11} PHYS	X_9 READ	X_4 SES	X_8 THINK	R^2
1. OLS	2.047	-.828	-1.162	-.299	.299	.070
2. Normalzn	.090	-.005	.123	-.026	.035	.064
3. Kasarda	.451	-.184	-.098	-.040	.089	.068
4. Hocking	.099	-.020	.129	.023	.038	.066
5. HK&B	.757	-.308	-.302	-.090	.117	.069
6. L. & W.	.098	-.018	.129	.023	.038	.066
7. H.K. Iter	.086	-.001	.119	.027	.035	.062
8. Vinod	.286	-.116	011	-.013	.073	.068
(3-values)	.088	-.003	.121	-.027	.035	.063
	.056	.028	.065	.028	.039	.038

a. The underlined coefficients are not statistically significant at the .05 level.

Table 8. Performance Indices of OLS and RR Methods for Equation 10 (CHEM) of the Science Achievement Model (N=1660, Dc=.992)

		Performance Indices					
Estimating Method	k	Vmax	Dmax	E.S.Ratio	RVAR	ϕ_o	α
1. OLS	0	51.14	.987	1.000	0	0	1.000
2. Normalization	.104	1.100	.063	2.90×10^{-5}	99.65%	.13%	.827
3. Kasarda	.014	11.36	.939	5.01×10^{-5}	73.45%	.03%	.990
4. Hocking	.009	17.05	.960	.015	61.43%	.02%	.996
5. H. K. & B.	.018	8.93	.920	2.59×10^{-3}	79.16%	.04%	.983
6. L. & W.	.011	13.99	.951	8.97×10^{-3}	67.48%	.02%	.994
7. H.K. Iterative	.032	4.75	.834	5.67×10^{-4}	88.54%	.06%	.958
8. Vinod	.089	1.26	.159	4.04×10^{-5}	96.14%	.12%	.855
(2-values)	1.08	.203	-.428	4.85×10^{-7}	99.61%	1.28%	.000

Table 9. Performance Indices of OLS and RR Methods for Equation 11 (PHYS) of the Science Achievement Model (N=1660, Dc=.997)

Estimating Method	k	Performance Indices Vmax	Dmax	E.S.Ratio	RVAR	ϕ_0	α
1. OLS	0	131.9	.995	1.000	0	0	1.000
2. Normalization	.103	1.58	.335	7.88×10^{-6}	98.26%	2.94%	.000
3. Kasarda	.001	92.42	.993	.246	26.31%	.05%	.972
4. Hocking et al.	.007	23.82	.972	7.83×10^{-3}	75.89%	.64%	.060
5. H. K. & B.	.001	101.6	.994	.342	20.13%	.03%	.991
6. L. & W.	.007	26.56	.975	9.86×10^{-3}	73.69%	.59%	.084
7. H. & K. Iterative	.001	95.52	.993	.275	24.21%	.04%	.980
8. Vinod	.060	2.19	.555	3.39×10^{-5}	97.16%	2.1%	.000
(2-values)	.831	.275	-.399	4.49×10^{-8}	99.77%	10.50%	.000

Table 10. Performance Indices of OLS and RR Methods for Equation 12 (SCATT) of the Science Achievement Model (N=1660. Dc=.999)

Estimating Method	k	Performance Indices Vmax	Dmax	E.S.Ratio	RVAR	ϕ_0	α
1. OLS	0	2,647.0	.9998	1.000	0	0	1.000
2. Normalization	.087	1.35	.212	8.29×10^{-6}	99.89%	.19%	.690
3. Kasarda	.001	96.69	.993	.032	95.33%	.10%	.887
4. Hocking et al.	.035	4.45	.820	5.89×10^{-5}	99.76%	.16%	.750
5. H. K. & B.	.0004	315.5	.998	.114	87.1%	.07%	.951
6. L. & W.	.040	3.78	.780	4.41×10^{-5}	99.78%	.16%	.745
7. H&K Iterative	.119	.971	-.184	4.23×10^{-6}	99.91%	.20%	.644
8. Vinod	.002	31.18	.979	8.94×10^{-3}	97.85%	.13%	.838
(3-values)	.099	1.12	.076	6.23×10^{-6}	99.90%	.19%	.673
	1.13	.195	-.432	3.49×10^{-8}	99.99%	.83%	.018

Table 11. Rankings of Performance Criteria

Performance Criteria	Normzn	Kas.	Hock.	HKB	L&W	HK It	Vin 1	Vin 2
k^a	7	3	1	4	2	5	6	8
$Vmaz^a$	1	6	8	5	7	4	2	3
$Dmax^a$	1	6	8	5	7	4	2	3
E.S.Ratio[b]	2	6	8	5	7	4	3	1
RVAR[c]	1	6	8	5	7	4	3	2
$\phi_0{}^a$	7	3	1	4	1	5	6	8
α^d	Rej	Acc	Acc	Acc	Acc	Rej	Rej	Rej

Table 11 a. Performance Criteria Ranking for Eq. 10 (CHEM)

Performance Criteria	Norman	Kas.	Hock.	HKB	L&W	HK It	Vin 1	Vin 2
k^a	7	1	4	1	4	1	6	8
$Vmax^a$	2	6	4	8	5	7	3	1
$Dmax^a$	2	6	4	8	5	7	3	1
E.S.Ratio[b]	2	6	4	8	5	7	3	1
RVAR[c]	2	6	4	8	5	7	3	1
$\phi_0{}^a$	2	6	4	8	5	7	3	1
α^d	Rej	Acc	Rej	Acc	Rej	Acc	Rej	Rej

Table 11 b. Performance Criteria Ranking for Eq. 11 (PHYS)

Table 11 (cont'd.)

Performance Criteria	Normzn	Kas.	Hock.	HKB	L&W	HK It	Vin 1	Vin 2	Vin 3
k[a]	6	2	4	1	5	8	3	7	9
Vmax[a]	3	8	6	9	5	2	7	1	4
Dmax[a]	3	8	6	9	5	2	7	1	4
E.S.Ratio[b]	4	9	6	8	5	2	7	3	1
RVAR[c]	4	8	6	9	5	2	7	3	1
ϕ_0[a]	8	2	4	1	4	7	3	6	9
α[d]	Rej	Rej	Rej	Acc	Rej	Rej	Rej	Rej	Rej

Table 11 c. Performance Criteria Ranking for Eq. 12 (SCATT)

Table 11 Footnotes:

a. Ranked from lowest (1) to highest (8).

b. Ranked from least sensitive to sampling error (1) to most sensitive (8).

c. Ranked from highest % reduction in variance (1) to lowest (8).

d. Whether the ridge estimate is α-acceptable (Acc) or not α-acceptable (Rej).